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Nonrelativistic Approximation of Scalar–Tensor Theory with Torsion and Intermediate-Range Force

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In the nonrelativistic approximation of the scalar–tensor theory with torsion, deviations from the Newtonian theory expressions of gravity, the gravitational spectral shift of light, and the stellar radius are derived.

1. INTRODUCTION

In recent years, the possibility of the existence of the intermediate-range force and its influence on Earth's gravity and stellar structure have been discussed by many authors. For example, the reanalysis of the EPF (Eötvös, Pékar, and Fekete) experimental data (Fischbach *et al.*, 1986) has suggested that there might be a deviation from the Newtonian inverse square law of the form

$$U(r) = -\frac{MG_\infty}{r}(1+\mu\, e^{-\lambda r}) \tag{1}$$

where μ and λ^{-1} are the strength and the range of the intermediate-range force, respectively. Stubbs *et al.* (1987) have given the constraint on μ. Measured by Holding *et al.* (1986), the deviation of the gravity residual in deep mines is as follows:

$$\Delta g_i = 4\pi G_\infty \rho\mu\left[Z - \frac{1}{2\lambda}(1-e^{-\lambda Z})\right] \tag{2}$$

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where Z is the depth. ρ is the density of the Earth. Measured by Eckhardt *et al.* (1988), the deviation of the gravity residual on a tower is

$$\Delta g_e = \frac{2\pi G_\infty \rho\mu}{\lambda}(e^{-\lambda H} - 1) \tag{3}$$

where H is the height.

In order to explain equation (1), O'Hanlon (1972) and Xu (1989) have suggested scalar–tensor theories in the Riemann spacetime V_4. Clearly, these can also explain equations (2) and (3). However, recently de Sabbata *et al.* (1990) and Xu *et al.* (1991) have suggested that the intermediate-range force be explained as a manifestation of torsion in the Riemann–Cartan spacetime U_4. They show only that, in the weak field linear approximation for a static point mass source, equation (1) is obtained from their theories. But they do not show clearly whether equations (2) and (3) can be derived from their theories. This paper further shows that, in the nonrelativistic approximation of the scalar–tensor theory with torsion (Xu *et al.*, 1991), equations (2) and (3) can also be derived. The deviations of the gravitational spectral shift of light and the stellar radius are found in this paper. Thus, in the next section, we briefly review the scalar–tensor model with torsion and the field equation. The weak field linear approximate solutions are given in Section 3. In Section 4, we calculate the gravitational spectral shift of light in the stellar interior and at the stellar exterior. In Section 5, we derive an expression for the deviation of the gravity residual from the equation of motion for a test particle. The Lane–Emden equation, modified to include the intermediate-range force, and the fractional change in the stellar radius are obtained in Section 6.

2. MODEL AND FIELD EQUATION

In the scalar–tensor model with torsion, the variational principle is (Xu *et al.*, 1991)

$$\delta \int [\varphi R + kL + \varepsilon(\varphi - \varphi_0)^2](-g)^{1/2}\, d^4x = 0 \tag{4}$$

where k is a constant, ε is a coupling parameter, φ is the scalar function, φ_0 is the constant background value for the scalar field φ, L is the Lagrangian density, which clearly does not include φ, for matter, and R is the curvature scalar in the Riemann–Cartan spacetime U_4 and can be written as follows:

$$R = R(\{\cdot\}) - 4S_{ij}{}^{j}S^{i}{}_{k}{}^{k} + 2S_{ijk}S^{ikj} + S_{ijk}S^{ijk} - \frac{4}{(-g)^{1/2}}((-g)^{1/2}S^{i}{}_{j}{}^{j})_{,i} \tag{5}$$

where $R(\{\cdot\})$ is the curvature scalar in the Riemann spacetime V_4, namely, the curvature scalar with respect to the Christoffel symbol. The comma "," indicates the usual derivative. $S_{ij}{}^{k}$ is the torsion tensor and is defined as

$$S_{ij}{}^{k}=\tfrac{1}{2}(\Gamma_{ij}{}^{k}-\Gamma_{ji}{}^{k}) \tag{6}$$

where $\Gamma_{ij}{}^{k}$ is the connection coefficient in U_4. Taking the torsion tensor as

$$S_{ij}{}^{k}=\frac{b}{2}\varphi^{-1}(\varphi_{,j}\delta_i^k-\varphi_{,i}\delta_j^k) \tag{7}$$

where b is a parameter which is independent of the spacetime point, then (5) becomes

$$R=R(\{\cdot\})-\omega\varphi^{-2}\varphi^{,k}\varphi_{,k}+\frac{6b}{(-g)^{1/2}}\varphi^{-1}(-g)^{1/2}\varphi^{,k})_{,k} \tag{8}$$

where $\omega=6b(b+1)$ is a new parameter. Substituting (8) into (4) and omitting the divergent term, we get

$$\delta\int[\varphi R(\{\cdot\})-\omega\varphi^{-1}\varphi^{,k}\varphi_{,k}+\varepsilon(\varphi-\varphi_0)^2+kL](-g)^{1/2}\,d^4x=0 \tag{9}$$

By varying g_{ij} and φ in (9), respectively, we obtain the field equations

$$\begin{aligned}G_{ij}(\{\cdot\})&=R_{ij}(\{\cdot\})-\tfrac{1}{2}g_{ij}R(\{\cdot\})\\&=\varphi^{-1}(\varphi_{,i|j}-g_{ij}\,\Box\varphi)+\omega\varphi^{-2}(\varphi_{,i}\varphi_{,j}-\tfrac{1}{2}g_{ij}\varphi^{,k}\varphi_{,k})\\&\quad+\tfrac{1}{2}\varepsilon g_{ij}\varphi^{-1}(\varphi-\varphi_0)^2+\tfrac{1}{2}k\varphi^{-1}T_{ij}\end{aligned} \tag{10}$$

$$\Box\varphi+\frac{2\varepsilon\varphi_0}{2\omega+3}(\varphi-\varphi_0)-\frac{k}{2(2\omega+3)}T=0 \tag{11}$$

where $R_{ij}(\{\cdot\})$ is the Ricci tensor with respect to the Christoffel symbol. $\Box\varphi=g^{ij}\varphi_{,i|j}$. The vertical bar symbol "|" denotes the covariant derivative using only the Christoffel symbol of the metric. According to the Bianchi identity, the Einstein tensor $G^{ij}(\{\cdot\})$ satisfies the following identity (Xu *et al.*, 1991)

$$G^{ij}(\{\cdot\})_{|j}=0 \tag{12}$$

The energy-momentum tensor of matter T_{ij} is defined as

$$T_{ij}=-\frac{2}{(-g)^{1/2}}\frac{\partial((-g)^{1/2}L)}{\partial g^{ij}} \tag{13}$$

$T=g^{ij}T_{ij}$. Using (10)–(12), we get that

$$T^{ij}{}_{|j}=0 \tag{14}$$

3. THE WEAK-FIELD LINEAR APPROXIMATE SOLUTIONS

For a weak field, we write

$$g_{ij} = \eta_{ij} + h_{ij}, \qquad \varphi = \varphi_0 + \xi \tag{15}$$

where η_{ij} is the Minkowskian metric tensor. h_{ij} and ξ are small perturbations and they are computed to the linear first approximation only. Therefore, substituting (15) into (11), we get

$$-\nabla^2 \xi + \frac{1}{c^2} \frac{\partial^2 \xi}{\partial t^2} + \lambda^2 \xi = \tfrac{1}{2} k \mu T \tag{16}$$

where $\lambda^2 = 2\varepsilon\varphi_0/(2\omega + 3)$ and $\mu = 1/(2\omega + 3)$. The retarded-time solution of equation (16) is

$$\xi = \frac{k\mu}{8\pi} \int \frac{T}{\gamma} e^{-\lambda r} \, d^3x \tag{17}$$

where T is to be evaluated at the retarded time. Substituting (15) into (10) and introducing the coordinate condition

$$(h_{ij} - \tfrac{1}{2} \eta_{ij} h)_{,k} \eta^{jk} = \varphi_0^{-1} \xi_{,i} \tag{18}$$

we find that equation (10) becomes

$$-\nabla^2 \alpha_{ij} + \frac{1}{c^2} \frac{\partial^2 \alpha_{ij}}{\partial t^2} = -k \varphi_0^{-1} T_{ij} \tag{19}$$

where

$$\alpha_{ij} = h_{ij} - \tfrac{1}{2} \eta_{ij} h - \eta_{ij} \varphi_0^{-1} \xi \tag{20}$$

The retarded-time solution of equation (19) is

$$\alpha_{ij} = -\frac{k\varphi_0^{-1}}{4\pi} \int \frac{T_{ij}}{r} \, d^3x \tag{21}$$

From equations (17), (20), and (21), we get

$$\begin{aligned} h_{ij} &= \alpha_{ij} - \tfrac{1}{2} \eta_{ij} \alpha - \eta_{ij} \varphi_0^{-1} \xi \\ &= \frac{k\varphi_0^{-1}}{4\pi} \left\{ -\int \frac{T_{ij}}{r} \, d^3x + \tfrac{1}{2} \eta_{ij} \int \frac{T}{r} (1 - \mu \, e^{-\lambda r}) \, d^3x \right\} \end{aligned} \tag{22}$$

We discuss two special cases as follows:

1. For a stationary mass point of mass M, from equations (15) and (22), we obtain the weak field approximate solutions

$$g_{44}=1+\frac{2U(r)}{c^2} \tag{23}$$

$$g_{\alpha\alpha}=-1-\frac{k\varphi_0^{-1}}{8\pi}\frac{c^2M}{r}(1-\mu\, e^{-\lambda r}) \qquad (\alpha=1,2,3) \tag{24}$$

where

$$U(r)=-\frac{kc^4M\varphi_0^{-1}}{16\pi r}(1+\mu\, e^{-\lambda r}) \tag{25}$$

putting $k=16\pi/c^4$ and with $\varphi_0^{-1}=G_\infty$ the Newtonian constant of gravitation for $r\to\infty$, then equation (25) becomes equation (1). Since $\lambda^{-1}\gg r$ for the mass separations in laboratory experiments, the constant of the laboratory derived from differentiating (1) to obtain gravity is

$$\begin{aligned} G_l &= G_\infty[1+\mu(1+\lambda r)\, e^{-\lambda r}] \\ &\approx G_\infty(1+\mu) \end{aligned} \tag{26}$$

2. For a static uniform sphere of perfect fluid with mass M, density ρ, and radius R, the nonzero components of the energy-momentum tensor are

$$T^\alpha_\beta=-p\delta^\alpha_\beta, \qquad T^4_4=\rho c^2 \qquad (\alpha,\beta=1,2,3) \tag{27}$$

Assuming that its pressure p is very small, i.e., $p\ll c^2\rho$, and substituting (27) into (22), we get the solutions in the interior of the sphere,

$$h_{44}=-\frac{8\pi G_\infty\rho}{c^2}\left[\tfrac{1}{2}(R^2-\tfrac{1}{3}r^2)-\frac{\mu}{\lambda^3 r}(\lambda R+1)\, e^{-\lambda R}\sinh(\lambda r)+\frac{\mu}{\lambda^2}\right] \tag{28}$$

$$h_{\alpha\alpha}=\frac{8\pi G_\infty\rho}{c^2}\left[\tfrac{1}{2}(R^2-\tfrac{1}{3}r^2)+\frac{\mu}{\lambda^3 r}(\lambda R+1)\, e^{-\lambda R}\sinh(\lambda r)-\frac{\mu}{\lambda^2}\right] \tag{29}$$

and the solutions at the exterior of the sphere,

$$h_{44}=-\frac{2G_\infty M}{c^2 r}-\frac{8\pi G_\infty\rho\mu}{c^2\lambda^3 r}\, e^{-\lambda r}[\lambda R\cosh(\lambda R)-\sinh(\lambda R)] \tag{30}$$

$$h_{\alpha\alpha}=\frac{2G_\infty M}{c^2 r}-\frac{8\pi G_\infty\rho\mu}{c^2\lambda^3 r}\, e^{-\lambda r}[\lambda R\cosh(\lambda R)-\sinh(\lambda R)] \tag{31}$$

4. THE GRAVITATIONAL SPECTRAL SHIFT OF LIGHT

In this section, we investigate the influence of the intermediate-range force on the gravitational spectral shift of light. The first case to be considered is the one in the stellar interior. The condition $\lambda R \gg 1$ is usually satisfied. Therefore in the stellar interior at the depth Z ($\ll R$) from the spherical surface, we obtain the approximate expression from equation (28) as follows:

$$h_{44}(R-Z) = -\frac{8\pi G_\infty \rho}{c^2}\left[\tfrac{1}{3}R^2 + \tfrac{1}{3}Z(R-\tfrac{1}{2}Z) + \frac{\mu}{\lambda^2}(1-\tfrac{1}{2}e^{-\lambda Z})\right] \quad (32)$$

At the stellar surface, we get

$$h_{44}(R) = -\frac{8\pi G_\infty \rho}{c^2}\left(\tfrac{1}{3}R^2 + \frac{\mu}{2\lambda^2}\right) \quad (33)$$

Thus, we find the gravitational spectral shift of light

$$\left(\frac{\delta \nu}{\nu}\right)_i = \frac{\nu(R-Z)-\nu(R)}{\nu(R)} = \left[\frac{1+h_{44}(R)}{1+h_{44}(R-Z)}\right]^{1/2} - 1$$

$$\approx \tfrac{1}{2}[h_{44}(R) - h_{44}(R-Z)]$$

$$\approx \left(\frac{\delta \nu}{\nu}\right)_N - \frac{G_l M \mu}{c^2 R^2} Z \quad (34)$$

where

$$\left(\frac{\delta \nu}{\nu}\right)_N = \frac{G_l M}{c^2 R^3} Z(R-\tfrac{1}{2}Z) \quad (35)$$

is the gravitational spectral shift of light in the Newtonian theory. The deviation of the gravitational spectral shift of light, due to the existence of the intermediate-range force, from the Newtonian theory is

$$\Delta\left(\frac{\delta \nu}{\nu}\right)_i = \left(\frac{\delta \nu}{\nu}\right)_i - \left(\frac{\delta \nu}{\nu}\right)_N$$

$$= -\frac{G_l M \mu}{c^2 R^2} Z \quad (36)$$

From equations (35) and (36), we get the relative deviation of the gravitational spectral shift in the stellar interior

$$I_1 = \frac{\Delta(\delta v/v)_i}{(\delta v/v)_N} = -\mu\left(\frac{R}{R-\frac{1}{2}Z}\right) \approx -\mu \tag{37}$$

In a similar manner, we discuss the situation external to the stellar surface. At the height H ($\ll R$) from the stellar surface, we obtain the approximate expression from equation (30) as follows:

$$h_{44}(R+H) \approx -\frac{2G_\infty M}{c^2 R} + \frac{2G_\infty M}{c^2 R^2} H - \frac{8\pi G_\infty \rho\mu}{c^2\lambda^2} e^{-\lambda H} \tag{38}$$

Thus we get the relative deviation of the gravitational spectral shift at the stellar exterior,

$$I_2 = \frac{\Delta(\delta v/v)_e}{(\delta v/v)_N} = \frac{3\mu}{2\lambda^2 RH}(1 - e^{-\lambda H}) \tag{39}$$

Therefore, comparing (37) and (39), we may predict that the deviation of the gravitational spectral shift of light according to general relativity will be greater in the stellar interior than at its exterior.

5. THE GRAVITATIONAL ACCELERATION

In this section, we find the deviation of the gravitational acceleration from the Newtonian theory arising from the intermediate-range force. The equation of motion for a test particle, derived by means of Papapetrou's method and equation (14), is

$$\frac{d^2x^i}{ds^2} + \begin{Bmatrix} i \\ jk \end{Bmatrix} U^j U^k = 0 \tag{40}$$

where $U^k = dx^k/ds$ is the 4-velocity of particle. Since the speed of the test particle is very much less than that of light, then $U^\alpha \approx 0$ ($\alpha = 1, 2, 3$), $U^4 = 1$. Thus, equation (40) becomes

$$\frac{d^2x^\alpha}{dt^2} + c^2 \begin{Bmatrix} \alpha \\ 44 \end{Bmatrix} = 0 \tag{41}$$

For the static weak field, equation (41) is written as

$$\frac{d^2x^\alpha}{dt^2} = -\frac{c^2}{2}\frac{\partial h_{44}}{\partial x^\alpha} \tag{42}$$

Substituting (28) into (42), we obtain the gravitational acceleration in the stellar interior

$$g_i(r) = -\frac{4\pi G_\infty \rho}{3} r + \frac{4\pi G_\infty \rho_\mu}{\lambda^3 r^2} (\lambda R + 1)\, e^{-\lambda R}[\sinh(\lambda r) - \lambda r \cosh(\lambda r)] \tag{43}$$

Substituting (30) into (42), we obtain the gravitational acceleration at the stellar exterior

$$g_e(r) = -\frac{G_\infty M}{r^2} - \frac{4\pi G_\infty \rho\mu}{\lambda^3 r^2} (1 + \lambda r)\, e^{-\lambda r}[\lambda R \cosh(\lambda R) - \sinh(\lambda R)] \tag{44}$$

The difference of the gravitational acceleration at the depth Z ($\ll R$) in the stellar interior and at the stellar surface is found approximately from equation (43) as follows:

$$\begin{aligned} \delta g_i &\equiv g_i(R - Z) - g_i(R) \\ &= \delta g_N - \frac{4\pi G_l \rho\mu}{1+\mu}\left[Z - \frac{1}{2\lambda}(1 - e^{-\lambda Z})\right] \end{aligned} \tag{45}$$

where

$$\delta g_N = \frac{2g_i(R)}{R} Z + 4\pi G_l \rho Z \tag{46}$$

is the difference of the gravitational acceleration in the Newtonian theory. Therefore, the deviation of the difference of the gravitational acceleration from the Newtonian theory in the stellar interior is

$$\begin{aligned} \Delta g_i &\equiv \delta g_i - \delta g_N \\ &= \frac{4\pi G_l \rho\mu}{1+\mu}\left[Z - \frac{1}{2\lambda}(1 - e^{-\lambda Z})\right] \end{aligned} \tag{47}$$

In a similar manner, we discuss the situation external to the stellar surface. From equation (44), we can find the expression of the deviation of the gravitational acceleration from the Newtonian theory exterior to the stellar surface,

$$\Delta g_e = \frac{2\pi G_l \rho\mu}{\lambda(1+\mu)} (e^{-\lambda H} - 1) \tag{48}$$

where H is the height from the stellar surface. Substituting (26) into (47) and (48), then we obtain equations (2) and (3), respectively.

As for the Earth, comparisons between the theoretical value of equations (47) and (48) and the experimental results are given by Holding *et al.* (1986) and Eckhardt *et al.* (1988).

6. THE LANE–EMDEN EQUATION

In this section, we study a static, spherically symmetrical perfect fluid with low pressure; its nonzero components of the energy-momentum tensor are

$$T^{\alpha}_{\beta} = -p(r)\delta^{\alpha}_{\beta}, \qquad T^{4}_{4} = \rho(r)c^2 \qquad (\alpha, \beta = 1, 2, 3) \tag{49}$$

Substituting (49) into (14), we obtain the equilibrium equation

$$p_{,\alpha} + \tfrac{1}{2}(p + \rho c^2)h_{44,\alpha} = 0 \tag{50}$$

Substituting (20) into (50), we get

$$\frac{1}{p+\rho c^2}\nabla p = -\tfrac{1}{2}(\nabla\alpha_{44} - \tfrac{1}{2}\eta_{44}\nabla\alpha - \eta_{44}\varphi_0^{-1}\nabla\xi) \tag{51}$$

where ∇ is the three-dimensional Laplacian operator. Taking the divergence, we get

$$\nabla \cdot \left(\frac{1}{p+\rho c^2}\nabla p\right) = -\tfrac{1}{2}\nabla^2\alpha_{44} + \tfrac{1}{4}\eta_{44}\nabla^2\alpha + \tfrac{1}{2}\eta_{44}\varphi_0^{-1}\nabla^2\xi \tag{52}$$

Substituting the static equations (16) and (19) into (52), taking account of the low-pressure approximation $p \ll \rho c^2$, and putting $K = 16\pi/c^4$ and $\varphi_0^{-1} = G_\infty$, we get

$$\nabla \cdot \left(\frac{1}{\rho}\nabla p\right) = -4\pi G_\infty \rho(1+\mu) + \tfrac{1}{2}G_\infty c^2\lambda^2\xi$$

or

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{1}{\rho}\frac{d}{dr}p\right) = -4\pi G_\infty \rho(1+\mu) + \tfrac{1}{2}G_\infty c^2\lambda^2\xi \tag{53}$$

We assume that the relationship between the pressure p and the density ρ is described by a polytropic equation

$$p = K\rho^{1+(1/N)} \tag{54}$$

where K is a constant. N is the polytropic index. Substituting (54) into (53) and introducing new variables

$$\vartheta = \left(\frac{\rho}{\rho_0}\right)^{1/N} \tag{55}$$

$$x = \left[\frac{4\pi G_\infty}{K(N+1)} \rho_0^{1-(1/N)}\right]^{1/2} r = \frac{r}{\beta} \tag{56}$$

where

$$\beta = \left[\frac{K(N+1)}{4\pi G_\infty} \rho_0^{(1/N)-1}\right]^{1/2} \tag{57}$$

we obtain the modified Lane–Emden equation

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2 \frac{d\vartheta}{dx}\right) = -(1+\mu)\vartheta^N + \frac{c^2\lambda^2}{8\pi\rho_0}\xi \tag{58}$$

where ρ_0 is the density at the center. The boundary conditions of (58) are

$$\vartheta(0) = 1, \qquad \frac{d\vartheta}{dx}(0) = 0 \tag{59}$$

In the absence of the intermediate-range force, then $\mu = 0$ and $\xi = 0$, and equation (58) becomes the Lane–Emden equation in the Newtonian theory.

For a static uniform star with density ρ_0 and radius R, we obtain the interior solution of equation (16),

$$\xi = \frac{8\pi\rho_0\mu}{c^2\lambda^3 r}\left[\lambda r - (1+\lambda R)\, e^{-\lambda R} \sinh(\lambda r)\right] \tag{60}$$

Therefore, the Lane–Emden equation (58) for $N = 0$ is written as

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2 \frac{d\vartheta}{dx}\right) + 1 = -\frac{\mu}{\lambda\beta x}(1+\lambda\beta x_0)\, e^{-\lambda\beta x_0} \sinh(\lambda\beta x) \tag{61}$$

where $x_0 = \beta^{-1}R$. This equation has the solution satisfying the conditions (59)

$$\vartheta(x) = 1 - \tfrac{1}{6}x^2 + \frac{\mu}{\lambda^2\beta^2}(1+\lambda\beta x_0)\, e^{-\lambda\beta x_0}\left[1 - \frac{1}{\lambda\beta x}\sinh(\lambda\beta x)\right] \tag{62}$$

Since the boundary condition at the stellar surface is $\vartheta(x_0)=0$ and taking the approximation $\lambda R \gg 1$, we obtain the expression of the stellar radius,

$$R = R_N\left(1-\frac{\mu}{2\lambda^2\beta^2}\right)^{1/2} \approx R_N\left(1-\frac{\mu}{4\lambda^2\beta^2}\right) \tag{63}$$

where $R_N=\sqrt{6}\beta$ is the stellar radius in the Newtonian theory. The fractional change in radius is, from equation (63),

$$\frac{\delta R}{R_N}=\frac{R-R_N}{R_N}=-\frac{\mu}{4\lambda^2\beta^2} \tag{64}$$

This result is the same as found by Glass *et al.* (1987) in another way.

7. CONCLUSION

Experimental measurements indicate the reliability of equations (1)–(3). Since equations (10) and (12) are satisfied in our theory, we can derive not only equation (1), but also equations (2) and (3) in the nonrelativistic approximation. Thus, our theory is consistent with experimental results.

In this paper, we have calculated the deviation of the gravitational spectral shift of light. This will provide another possible way to test the existence of the intermediate-range force.

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